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A comparative study of effect of complex conjugate terms in ring laser cavity

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ABSTRACT

The frequency splitting arises from the removal of the mode degeneracy existing for the two oppositely traveling waves, due to the differential cavity path length change produced by the rotation. We have derived gain and dispersion relations using real and then complex conjugate terms of electric field and polarization in Maxwell's equation. It is observed that the new equations are different from the original equations derived by Lamb and his coworkers and have physical significance. In this case mode pulling is also affected.

Key words: Mode degeneracy**,** cavity, cavity path length, gain, dispersion, ring cavity.

INTRODUCTION

A ring laser [1] can be constructed by three or four mirrors. They form a close light path in the shape of triangle or square. This assembly is rotated and the cavity has different effective lengths for the two directions of propagation. This results in a splitting of the cavity [2] frequencies which is proportional to the speed of rotation. Lamb and his coworkers [3-7] have successfully derived different parameters of laser cavities with the help of semiclassical theory of laser. The purpose of the present work is to present a comparison [8] of gain and dispersion relations derived using real and complex conjugate terms.

MATERIALS AND METHODS

In the theoretical model, we considered the basic equations of the semiclassical theory of laser where the cavity is primarily a two- mirror system. The analysis can also be written to cover the three mirror ring laser which may be constructed to support a single, clockwise running wave characterized by the normal mode function

$$U_n(z) = \exp(i K_n z) \quad , K_n = n\frac{\pi}{L}$$

The electric field in a ring laser E (z, t) can be written as the superposition of two waves traveling in opposite directions i.e., sines and cosines. An equivalent and more convenient representation consists of oppositely directed running waves are given by,

$$E(z,t) = \frac{1}{2}\{E_{+n}(t)\exp[-i(\nu_{+n}t + \phi_{+n} - K_{+n}z)] + E_{-n}(t)\exp[-i(\nu_{-n}t + \phi_{-n} - K_{-n}z)]\} + c.c.$$

Here, the amplitudes and phases are assumed to be slowly varying. For simplicity, we consider the two oppositely directed waves with approximately equal frequencies $(\nu_{+n} \approx \nu_{-n})$ and wave numbers $(K_{+n} = K_{-n})$. The electric field reduces to

$$E(z,t) = \frac{1}{2}\{E_{+}\exp[-i(\nu_{+}t + \phi_{+} - K_{+}z)] + E_{-}\exp[-i(\nu_{-}t + \phi_{-} - K_{-}z)]\} + c.c. \quad (1)$$

For traveling wave modes, the mode index n is negative for modes traveling in negative z-direction. With the corresponding induced polarization

$$P(z,t) = \frac{1}{2}\{P_{+}\exp[-i(\nu_{+}t + \phi_{+} - K_{+}z)] + P_{-}\exp[-i(\nu_{-}t + \phi_{-} - K_{-}z)]\} + c.c. \quad (2)$$

The Maxwell's equations in mks unit as, neglecting vector properties

$$\frac{\partial^2 E(z,t)}{\partial t^2} + \mu_0 \sigma \frac{\partial E(z,t)}{\partial t} + \mu_0 \varepsilon_0 \frac{\partial^2 E(z,t)}{\partial t^2} = -\mu_0 \frac{\partial^2 P(z,t)}{\partial t^2} \quad (3)$$

Here P, polarization is used to describe the induced atomic polarization of the active medium. It is desirable to provide for different cavity resonant frequencies for the linearly polarized radiation along orthogonal Cartesian axes transverse to the maser axis.

By using equations (1) and (2) only with real terms in equation (1) and as in the model, the medium is assumed to be dilute and the losses are small and $E_{+}, \phi_{+}, P_{+}, E_{-}, \phi_{-}, P_{-}$ also very small in optical frequency range, therefore, we can neglect terms containing $\ddot{E}_{+}, \ddot{E}_{-}, \ddot{P}_{+}, \ddot{P}_{-}, \dot{E}_{+}, \dot{E}_{-}, \dot{\phi}_{+}, \dot{\phi}_{-}, \sigma\dot{E}_{+}, \sigma\dot{E}_{-}, \dot{\phi}_{+}P_{+}, \dot{\phi}_{-}P_{-}, \dot{P}_{+} \, and \, \dot{P}_{-}$ the wave equation reduces to

$$\begin{aligned}
&\Rightarrow \{\Omega_{+}^2 E_{+} - i\left(\frac{\sigma}{\varepsilon_0}\right)\nu_{+}E_{+} - 2i\nu_{+}\dot{E}_{+} - (\nu_{+} + \dot{\phi}_{+})^2 \dot{E}_{+}\}\exp[-i(\nu_{+}t + \phi_{+} - K_{+}z)] \\
&\quad + \{\Omega_{-}^2 E_{-} - i\left(\frac{\sigma}{\varepsilon_0}\right)\nu_{-}E_{-} - 2i\nu_{-}\dot{E}_{-} - (\nu_{-} + \dot{\phi}_{-})^2 E_{-}\}\exp[-i(\nu_{-}t + \phi_{-} + K_{-}z)] \\
&= \nu_{+}^2 \varepsilon_0^{-1} P_{+}\exp[-i(\nu_{+}t + \phi_{+} - K_{+}z)] + \nu_{-}^2 \varepsilon_0^{-1} P_{-}\exp[-i(\nu_{-}t + \phi_{-} + K_{-}z)]
\end{aligned}$$

Where, $\Omega_{+} = K_{+}c \qquad \Omega_{-} = K_{-}c \qquad and \qquad c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$.

Now, putting $\sigma = \varepsilon_0 \frac{\nu_{+}}{Q_{+}}$ and taking $\nu = \nu_{\pm}$

$$\begin{aligned}
&\Rightarrow \{(\Omega_{+} - \nu_{+} - \dot{\phi}_{+})^2 E_{+} - \frac{1}{2} i \frac{\nu_{+}}{Q_{+}} E_{+} - i\dot{E}_{+}\}\exp[-i(\nu_{+}t + \phi_{+} - K_{+}z)] \\
&\quad + \{(\Omega_{-} - \nu_{-} - \dot{\phi}_{-})^2 E_{-} - \frac{1}{2} i \frac{\nu_{-}}{Q_{-}} E_{-} - i\dot{E}_{-}\}\exp[-i(\nu_{-}t + \phi_{-} + K_{-}z)] \\
&= \frac{1}{2}\nu_{+}\varepsilon_0^{-1} P_{+}\exp[-i(\nu_{+}t + \phi_{+} - K_{+}z)] + \frac{1}{2}\nu_{-}\varepsilon_0^{-1} P_{-}\exp[-i(\nu_{-}t + \phi_{-} + K_{-}z)] \quad (4)
\end{aligned}$$

Equating real and imaginary parts of equation (4) and taking two oppositely directed waves (which are conveniently represented by '+' and '-'sign) separately, we get the self- consistency equations for ring laser as given below:

$$\dot{E}_{+} + \frac{1}{2}\frac{\nu_{+}}{Q_{+}}E_{+} = -\frac{1}{2}\frac{\nu}{\varepsilon_0}\text{Im Part of } (P_{+}) \qquad (5)$$

$$\nu_{+} + \dot{\phi}_{+} = \Omega_{+} - \frac{1}{2}\frac{\nu_{+}}{\varepsilon_0}E_{+}^{-1}\text{Re part of } (P_{+}) \qquad (6)$$

$$\dot{E}_{-} + \frac{1}{2}\frac{\nu_{-}}{Q_{-}}E_{-} = \frac{1}{2}\frac{\nu_{-}}{\varepsilon_0}\text{Im part of } (P_{-}) \qquad (7)$$

$$\nu_{-} + \dot{\phi}_{-} = \Omega_{-} - \frac{1}{2}\frac{\nu_{-}}{\varepsilon_0}E_{-}^{-1}\text{Re part of } (P_{-}) \qquad (8)$$

Here, the possibility of a rotating frame is allowed by taking $\Omega_{+} \neq \Omega_{-}$.

Equations (5) through (7) determine the field amplitudes and frequencies for ring laser, once $P_{\pm}(t)$ are known in terms of the $E_{\pm}(t)$. To evaluate $P_{+}(t)$, we need a quantum description of the atomic medium.

These equations are derived without using the complex conjugate terms in the expressions for electric field and polarizations.

By using the complex conjugate terms in Maxwell's equation, we get the self- consistency equations as follows:

$$\dot{E}_{+} + \frac{1}{2}\frac{\nu_{+}}{Q_{+}}E_{+} = -\frac{1}{2}\frac{\nu}{\varepsilon_0}\text{Im Part of } (P_{+}) \qquad (9)$$

$$(2\nu_{+} + \frac{\Omega_{+}}{\nu_{+}}\dot{\phi}_{+}) = \Omega_{+} - \frac{1}{2}\frac{\nu_{+}}{\varepsilon_0}E_{+}^{-1}\text{Re part of } (P_{+}) \qquad (10)$$

Also

$$\dot{E}_{-} + \frac{1}{2}\frac{\nu_{-}}{Q_{-}}E_{-} = -\frac{1}{2}\frac{\nu_{-}}{\varepsilon_0}\text{Im part of } (P_{-}) \qquad (11)$$

$$(2\nu_{-} + \frac{\Omega_{-}}{\nu_{-}}\dot{\phi}_{-}) = \Omega_{-} - \frac{1}{2}\frac{\nu_{-}}{\varepsilon_0}\text{Re part of } (P_{-}) \qquad (12)$$

RESULTS AND DISCUSSION

Thus using complex conjugate terms we get the four self-consistency equations of which equations (9) and (11) are same as given by equations (5) and (7). However in place of equation (6), we get equation (10) and also in place of equation (8), we get equation (12).

Now, from equation (10)

$$(2\nu_{+} + \frac{\Omega_{+}}{\nu_{+}}\dot{\phi}_{+}) = \Omega_{+} - \frac{1}{2}\frac{\nu_{+}}{\varepsilon_0}E_{+}^{-1}\text{Re part of } (P_{+})$$

$$\Rightarrow [\nu_{+} + \nu_{+} + \dot{\phi}_{+} - (\dot{\phi}_{+} + \frac{\Omega_{+}}{\nu_{+}}\dot{\phi}_{+})] = \Omega_{+} + \frac{1}{2}\frac{\nu_{+}}{\varepsilon_0}E_{+}^{-1}\text{Re}\,of\ (P_{+}) - \frac{1}{2}\frac{\nu_{+}}{\varepsilon_0}E_{+}^{-1}\text{Re}\,of\ (P_{+}) - \frac{1}{2}\frac{\nu_{+}}{\varepsilon_0}E_{+}^{-1}\text{Re}\,of\ (P_{+}) \qquad (13)$$

Equation (13) has two parts. The first part may be written as

$$\nu_+ + \dot{\phi}_+ = \Omega_+ - \frac{1}{2}\frac{\nu_+}{\varepsilon_0} E_+^{-1} \operatorname{Re} of \ (P_+) \quad (14)$$

Equation (14) is same as equation (6). The second part of the equation is

$$\nu_+ - (\dot{\phi}_+ + \frac{\Omega_+}{\nu_+}\dot{\phi}_+) = \frac{1}{2}\frac{\nu_+}{\varepsilon_0} E_+^{-1} \operatorname{Re} of \ (P_+) - \frac{1}{2}\frac{\nu_+}{\varepsilon_0} E_+^{-1} \operatorname{Re} of \ (P_+)$$

$$\Rightarrow \nu_+ - \dot{\phi}_+ = \frac{\Omega_+}{\nu_+}\dot{\phi}_+ \quad (15)$$

CONCLUSION

The equation (12) also gives rise to similar equations. Thus we observe that introduction of the complex conjugate term affects on the frequency determining equations which represents dispersion in the medium. In this case mode pulling is affected. Thus it is observed that the complex conjugate terms have overall affect on the gain and dispersion relations.

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