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A common fixed point theorem for a pair of mappings in dislocated metric spaces

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ABSTRACT

In this paper we have proved fixed point theorem for continuous contraction mappings in dislocated Quasi Metric Spaces. Also we obtain a common fixed point theorem for a pair of mappings in Dislocated Metric Spaces. The purpose of this paper is to prove some fixed point theorems satisfying rational type of contractive condition due to Jaggi [1] in the setting of dislocated-quasi metric spaces. Also for A-Contraction, a general class of contraction defined by Akram et al. [2], established a common fixed point theorem for a pair of mappings in dislocated metric space.

Keywords: Dislocated quasi matrices fixed point, Continous contraction mapping

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INTRODUCTION

Dass and Gupta [1] generalized Banach's Contraction Principle in Metric Space. Also Rhoads [1977] introduced a partial ordering for various Definitions Contractive Mappings. The objective of this theorem is to prove some fixed point theorem for continuous contraction mapping defined by Dass and Gupta [1] , Rhoades [4]and Banach in Dislocated Quasi Metric Spaces. Banach [1992] proved Fixed Point Theorem for Contraction Mappings in Complete Matrix Space. It is well known as a Banach Fixed Point Theorem.

PRELIMINARIES

Definition 1 [3] : A Sequence $[X_n]$ is dq Metric Space (Dislocated Quasi Metric Spaces) (X, d) is called Cauchy Sequence if for given $\varepsilon > 0, \exists\ a\ n_0 \in N$ such that $\forall\ m, n > n_0 \Rightarrow d(x_m, x_n)$

$< \varepsilon$ or $d(x_n, x_m) < \varepsilon$

i.e. $min\,\{d(x_m, x_n),\ d(x_n, x_m)\} < \varepsilon$

Definition 2 [3] : A Sequence $[X_n]$ dislocated Quasi Convergence to x if

$1t\ n \to \infty\ d(x, x_n) = 1t\ n \to \infty\ d(x, x_n) = 0$

In this case x is called a dq limit of $[X_n]$ we write $x_n \to x$

Definition 4 [3] : A *dq* Metric Space (*X*, *d*) is called complete if every Cauchy Sequence in it is a *dq* convergent.

Definition 5 [3] : Let (*X*, *d*) and (*Y*, *d*) be *dq* Metric Spaces and Let $f: X \to Y$ be a function. Then f is which is $d_1 - q$ convergent to x_0 in X, the sequence $[f(x_n)]$ is d, -q convergent to x_0 in X, the sequence $[f(x_n)]$ is d, -q convergent to $f(x_0)$ in *Y*.

Definition 6 [3] : Let (*X*, *d*) be a *dq* Metric Space. A map $f: X \to X$ is called contraction if there exists $0 < x < 1$ such that

$$d(Fx, Fy) < \lambda\, d(x, y) \forall x, y \in X$$

Theorem 1 : Let (*X*, *d*) be a *dq* Metric and let $f: X \to X$ be continuous contracting mapping. Then F has a unique fixed point.

RESULTS

Theorem 1 : Let (*X*, *d*) be a *dq* Metric Space and let $f: X \to X$ be continuous mapping satisfying the following condition.

$$d(Fx, Fy) = \alpha \frac{(d(y, Fy)[1 + d(x, Fx)]}{(d(x, Fx)[1 + d(y, Fy)]}$$

$$\forall\, x, y \in X, \quad \alpha > 0, \quad \beta > 0, \quad r > 0, \quad \alpha + \beta + \gamma < 1$$

Then F has a unique fixed point.

Proof : Let $[X_n]$ be a sequence in X defined as follows. Let

$$x_0 \in X, \quad F(x_0) = x_1, \quad F(x_1) = x_2, \quad F(x_3) = x_4 \ldots\ldots\ldots$$

$$F(x_n) = x_{n+1}$$

Consider

$$d(x_n, x_{n+1}) = d(Fx_{n-1}, Fx_n) <$$

$$\alpha \frac{d(x_n, Fx_n)[1 + d(x_{n-1}, F x_{n-1})]}{[1 + d(x_{n-1}, x_n)]} + \beta d(x_{n-1}, x_n) + \gamma d(x_n, Tx_n)] \ldots\ldots\ldots\ldots (i)$$

$$d(x_n, x_{n+1}) < \alpha \frac{d(x_n)}{[1+d(x_{n-1}, x_n)]} + \beta d(x_{n-1}, x_n) + \gamma d(x_n, x_{n+1})$$

Therefore
$$d(x_n, x_{n+1}) - \alpha d(x_n, x_{n+1}) - \gamma d(x_n, x_{n+1})$$

$$\Rightarrow d(x_n, x_{n+1}) < \frac{\beta}{1 - \alpha - \gamma} d(x_{n-1}, x_n)$$
Let $\delta = \frac{\beta}{1-\alpha-\gamma}$ $with\ 0 < \delta < 1$
Then $d(x_n, x_{n+1}) < \delta\, d(x_{n-1}, x_n)$

on further decomposing we get

$$d(x_{n-1}, x_n) < \delta\, d(x_{n-2}, x_{n-1})$$

and finally we can write

$d(x_n, x_{n+1}) < \delta\, d(x_{n-2}, x_{n-1})$

On continuing this process n times

Since $0 < \delta < 1$ and $n \to \infty, \delta^n \to 0$.

Hece $[X_n]$ is a dq sequence in the complete dislocated Quasi Metric Space X.

Thus $[X_n]$ dislocated Quasi sequence converges to come t_0. Since F is continuous we have

$F(t_0) = 1t_{n\to\infty} F(X_n) = 1t_{n\to\infty}\ X_{n+1} = t_0$

Thus $F(t_0) = t_0$

Thus F has a fixed point.

Uniqueness
Let x be a fixed point of F. Then by given condition we have

$d(x,x) = d(Fx,x) < (\alpha + \beta + \gamma)d(x,x)$

Which gives $d(x,x) = 0$ Since

$0 < (\alpha + \beta + \gamma) < 1$ and $d(x,x) > 0$.

Thus $d(x, Fx) = \mathrm{d(x,x)}$

if x is a fixed point of F.

Let $x, y \in X$ be fixed point of F, i.e. is $Fx = x; Fy = x; Fy = y$

Then by condition 3.1 $d(x,y) = d(Fx,Fy) < \beta\, d(x,y)$

Similarly $d\,(y,x) = 0$ and hence $x = y$. Thus fixed point of F is unique.

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